Different Approaches to Inductance Calculation of E-I Gapped Iron Core Inductor

Abstract. In the paper inductance calculation by using different methods is presented. The object of study is an E-I gapped iron core inductor. The calculation is started with classical analytical approach and by using magnetic circuit theory (MCT); later on it is continued with another three numerical approaches, done via Finite Element Method (FEM). The results of calculations are compared with the measured values of inductance. The evaluation of different methodologies is presented.

Introduction

Iron core coils, also called inductors or reactors, are used for various purposes in electrical engineering practice. Depending on the power, inductors are performed as single phase, or three phases. Being more economically, they are usually made with an iron core. In practical use, the most frequently is found a shell-type E-I shaped core. The value of magnetizing inductance is one of the most important features of iron core inductor. In order to adjust the requested value of the inductor inductance, a changeable analysed structure. Depending on the requested value of influencing to the magnetising and circuit parameters of the can be adjusted in the range from 0.4 mm up to 1.0 mm. possibility is to change the depth of the magnetic core, number of turns of the excitation winding. Another through a magnetic circuit approach. Evaluation of the is obtained by operating on the air-gap length and on the working and loading conditions [1]. The inductance variation of the inductor inductance, a changeable air-gap is always anticipated.

Either for designers or for users, it is always an interesting issue how to calculate the magnetizing inductance value, as accurate as possible. Although the inductance can be easily estimated analytically by using simple magnetic circuit theory, this approach typically is not accurate as ignores flux leakage and fringing effects. To resolve these effects in more details, a numerical method by using the Finite Element Analysis is applied. The goal of the present paper is to show how inductance is calculated by using various Finite Element Method (FEM) simulations and to compare these results to the approximation obtained through a magnetic circuit approach. Evaluation of the different approaches, with respect to the accessibility and accuracy, is done on the basis of the testing results.

Example Geometry

The studied object is a single phase variable inductance iron core inductor, connected to street lighting lamps; the apparent and active power are variable, depending on the working and loading conditions [1]. The inductance variation is obtained by operating on the air-gap length and on the number of turns of the excitation winding. Another possibility is to change the depth of the magnetic core, influencing to the magnetising and circuit parameters of the analysed structure. Depending on the requested value of the inductor reactance, the operating length of the air gap can be adjusted in the range from 0.4 mm up to 1.0 mm. The example geometry is composed of an E-I gapped laminated core with a rectangular cross-section extending for 36 mm in the into-the-page direction. The iron core is shell-type formed by silicon-steel laminations of magnetic material type EN 230-50, having the relative permeability μ_r=1630. The rated working value of the air-gap length is 0.815 mm. The I-part is freely displacing, while excitation winding lies in the slots of the fixed E-part of the inductor and is consisted of 608 copper wire turns with heavy build insulation; the copper fill fraction in the coil window is 49%.

In accordance with the application to which is assigned the considered inductor, the rated data are: voltage supply \( U_n = 220 \, \text{V} @ 50 \, \text{Hz} \); current \( I_n = 1.15 \, \text{A} \); power \( P_n = 125 \, \text{W} \) and power factor \( \cos \varphi_n = 0.55 \). The starting voltage of firing the lamp is prescribed to be \( U_{\text{min}} = 160 \, \text{V} \). The front view of the gapped iron core inductor, as well as its geometrical cross-section is shown in Figure 1, (a) and (b) respectively.

![Fig.1. Studied E-I gapped iron core inductor](image)

Problem Definition

The aim of the study is determination the value of the magnetising inductance of an E-I gapped iron core inductor (ICI). The main goal is to present and to apply different approaches to calculation of this important electromagnetic quantity and to discuss both their accuracy and availability. In the paper, there will be presented the methodology for calculation of the leakage inductance, as well; this quantity combined with the magnetising inductance is enabling the possibility to determine the main inductance of the iron core inductor. The value of the main inductance is defining the inductor's main reactance; consequently, it is determining the behaviour of the device, and in particular, the power factor and the reactive energy consumption of the inductor.

In the electromagnetic devices where the variation of inductance is influenced by the variation of the air-gap length, it is always very important to investigate and study their mutual dependence. It is obvious that magnetising inductance of the gapped iron core inductor is dependent on the air-gap length. In accordance to the methodology of each particular approach, it is comprehended to carry out several calculations and/or analyses; thus, collecting and combining the results, instead of calculation only one value, the whole characteristics will be derived.
Furthermore, if Finite Element Method (FEM) is used, it is also possible to assess the attractive forces between the movable and fixed part of the analysed inductor.

**Leakage Inductance Calculation**

The power factor of the inductor is dependent on the main reactance \( X \), determined by the main inductance \( L \). The inductance \( L \) of the iron core inductor is consisted of two main parts: the leakage inductance \( L_s \) of the coil (winding), usually constant and non depending on current value; the magnetising inductance \( L_m \), usually depending on the saturation level of the iron core, and changing nonlinear with the excitation current.

\[
(1) \quad L = L_s + L_m
\]

For calculation of the leakage inductance, we are using the existing analytical procedures [2]. Introducing the expression for detailed analytical calculation of the leakage inductance and having available the exact geometrical dimensions of the studied E-I core structure, the winding leakage inductance is determined to be:

\[
(2) \quad L_s = 0.1198 \, [\text{H}]
\]

This value of the leakage inductance will be used later on, when splitting the components of the main inductance.

**Magnetizing Inductance Calculation – Analytical Approach**

Calculation of magnetizing inductance is always more complicated nonlinear problem, and different approaches both analytical and numerical, can be used. Some of them are more exact, but more complicated for practical use; another are simple to apply, but not give enough accurate results. The intention of this paper is to present several different approaches to inductance calculation and to compare the obtained results. The appraisal of calculated results will be given on the basis of the measured values.

The brief description of the applied methods and the computational results are given in continuation.

**By using Flux Linkage – Linear case**

As a guideline and first step, a rough approximation of inductance value is done at working condition with minimum value of the voltage supply \( U_{min}=160 \text{V} \), and with starting current in the coil \( I_s=1.35 \text{A} \). Presuming the linear magnetic core, and adopting for winding voltage drop an average value of 15\%, one can calculate the winding flux linkage from the expression:

\[
(3) \quad \Psi_n = N \Phi_n = \frac{E}{4.44 f_n} = \frac{0.85 U_{min}}{4.44 f_n} = L \cdot I_n
\]

where: \( N=608 \) is number of turns of the winding; The main inductance \( L \) is determined to be:

\[
(4) \quad L = 0.4538 \, [\text{H}]
\]

The magnetising inductance is easily found, by using the value of the leakage inductance from Eq. (2):

\[
(5) \quad L_m = 0.3340 \, [\text{H}]
\]

**By using Flux Linkage – Non Linear case**

The previous result is predicted by introducing several assumptions and this simple approach tends to raise the value of magnetizing inductance. First, the voltage drop of 15\% is only tentative value. Second, even the voltage is with a sinusoidal wave form, the magnetising current is significantly distorted; applying the fundamental harmonic for current, we do not take into account the higher level saturation of the magnetic core. Obviously, this value given by Eq. (5) can be accepted only as the first approximation quantity, and should be exacted. In Eq. (3) it is introduced a correction coefficient for the non sinusoidal current wave.

\[
(6) \quad L = L_m + L_s = \frac{\Psi_n}{I} = \frac{N \Phi_n}{\nu I_n}
\]

where: \( \nu=\nu_s \) is non-sinusoidal magnetising current, defined by a distortion \( \nu \) and rated current \( I_n \).

The current distortion \( \nu \) is computed by improving the analytical procedure [3]. To take into account saturation of the core, we suggest applying an iterative procedure. This approach is recommended to be used by the designers, when developing a new inductor [1]. The procedure is carried out iteratively in several steps; it takes 4-5 iterations to get equal the presumed (initial) and the calculated (final) value of induced voltage \( E \), giving the accurate value of the magnetic flux density \( b \) in the core. Afterwards, one can found for \( \nu=1.171 \) and the main inductance is calculated,

\[
(7) \quad L = 0.4035 \, [\text{H}]
\]

while the magnetising inductance will be:

\[
(8) \quad L_m = 0.2837 \, [\text{H}]
\]

**By using Magnetic Circuit Theory – MCT**

A more elaborated analytical circuit model, based on the magnetic circuit theory of the E-I gapped iron core inductor (ICI) is developed [1,4]. It is possible to take into account the magnetic reluctance of the core.

At the beginning as a guideline, an analytical expression for calculation of the inductance is derived. In the first approximation we can easy assume that there is no fringing, but we do not neglect the contribution of the iron sections, as usually the magnetic core is saturated and the gap length is relatively small. From the study of magnetic circuit, the following expression is displayed:

\[
(9) \quad I_m = \frac{\mu_0 N^2}{g + \frac{l_{Fe,av}}{S_g \mu_s S_{Fe}}}
\]

where: \( g^2=2\pi \) – total length of the air-gap along a flux line; \( S_g \) – cross section of the air-gap; \( S_{Fe} \) – cross section of the iron core; \( l_{Fe,av} \) – average flux line length; \( \mu_s=1650 \) – relative permeability of the core material.

If we consider only the air-gap magnetic field, then we derive a simpler expression:

\[
(10) \quad I_m \approx N^2 \frac{\mu_0 S_g}{g}
\]

The inductance is essentially inversely proportional to the air-gap length. Thus, constraints imposed on the variation of the air-gap correspond to constraints on the variation of the inductance itself. The variation of operating air-gap length \( g \) of the ICI is between 0.4 and 1.0 mm.

All geometrical dimensions of the analysed E-I gapped iron core inductor, having the topology displayed in Fig. 1 (b), are known exactly. As the analyzed structure is
characterised by a variable air-gap, the calculation of the magnetising inductance is performed at a working length $g_{av} = 0.815\,\text{mm}$ and the result is:

\[
L_m = 0.2703 \, [\text{H}]
\]

**Magnetizing Inductance Calculation – Numerical FEM Approach**

When nonlinear materials are involved in the analysed geometry and the core is saturated even at a presence of air gaps, the calculations get somewhat more complicated. In order to take into consideration all electromagnetic phenomena, it is applied Finite Element Method (FEM) for calculation magnetic field distribution of iron core inductor. It is also possible to take into account secondary effects: an extra flux component along fringing paths crossing the gaps (outer and inner) and leakage in the coil windows.

**FEM Calculation of the Magnetic Field**

Numerical magnetic field solution is performed running FEMM [5]. The detailed FEM model of the E-I inductor is developed; the exact geometry, boundary conditions and all material properties are carefully built-in the FEM model of the studied structure. A fairly fine mesh, especially in the air-gap is anticipated; at maximum length of 1.0 mm it is consisted of 23,955 nodes and 47,648 elements.

As an example, in Fig. 2 is presented magnetic field distribution at rated current $I_n = 1.15\,\text{A}$, for the following gaps: (a) $g_{min} = 0.4\,\text{mm}$; (b) $g_{av} = 0.815\,\text{mm}$; (c) $g_{max} = 1.0\,\text{mm}$. In order to enable an evident comparison of the field strength, it is adopted the same scale of the flux density intensity and the same difference of magnetic vector potential value $A$ between two adjacent flux lines; consequently, each figure is characterised with different number of flux lines.

In addition, it is analysed flux density distribution along the middle of the air gap. The characteristics at the rated excitation current and three typical air-gap length are given in Fig. 3 (a), (b) and (c), in the same way as previously.

![Magnetic field distribution in E-I gapped iron core inductor](image)

**Fig.2. Magnetic field distribution in E-I gapped iron core inductor**

- (a) $g_{min} = 0.4\,\text{mm}; A_{max} = 0.0135\,\text{Vs/m}; n = 33\,\text{lines}$
- (b) $g_{av} = 0.815\,\text{mm}; A_{max} = 0.0083\,\text{Vs/m}; n = 21\,\text{lines}$
- (c) $g_{max} = 1.0\,\text{mm}; A_{max} = 0.0073\,\text{Vs/m}; n = 19\,\text{lines}$

**Fig.3. Magnetic flux density along the mid-gap line of the middle leg**

- (a) $g_{min} = 0.4\,\text{mm}; B_{mid} = 0.8514\,\text{T}$
- (b) $g_{av} = 0.815\,\text{mm}; B_{mid} = 0.4816\,\text{T}$
- (c) $g_{max} = 1.0\,\text{mm}; B_{mid} = 0.4141\,\text{T}$
Finite Element Magnetizing Inductance

The finite element solutions can be used directly to find inductance [4,6]. There are a number of well established numerical methods of finding inductance and generally speaking all work very well. The values of inductance found from FEM computations generally agree well with measured results. Results of inductance calculation converge quickly with decreasing mesh size. Consequently, when analysing particular attention is paid on the air-gap region, where the main change of the magnetic occurrences takes place.

In Fig. 4 is presented mesh of the section influencing the most on the magnetizing inductance value. The shown section is taken over the middle leg of the inductor; the air-gap mesh is piled between two blue lines; the straight black line is the mid-gap line to which the Fig. 3 is corresponding.

Fig. 4. A part of the air-gap mesh at $g_{max} = 1.0$ mm

Depending of the analysed problem, one can compute either apparent or differential magnetising inductance [4]; both are functions of the air-gap length $g$. On the other hand, due to the nonlinear characteristic of ferromagnetic materials of the core, the inductances are also functions of the currents $i$ flowing through the excitation coil.

In general, the apparent inductance is defined at a particular air-gap length $g$, considering the operating point $(i_o,\lambda_o)$ of the iron core inductor. If the excitation current is $i_o$, while the flux linkage is $\lambda_o$, the apparent inductance is determined by the relation:

$$L_{app}(i_o, g_o) = \frac{\lambda_o}{i_o}$$

where: $i_o$ – current in the excitation winding; $\lambda_o$ – flux linkage of the coil, at the working point.

The ratio between an infinitesimal variation of flux linkage $d\lambda$, and the corresponding variation of current $di$, around the same operating point $(i_o,\lambda_o)$, defines the differential inductance of the ICI, which is:

$$L_{dif}(i_o, g_o) = \frac{d\lambda_o}{di_o}(i_o,\lambda_o)$$

In linear conditions, since the permeability of the core material is constant, the two values obtained from the last two equations are equal: $L_{app} = L_{dif} = L_{m}$.

Alternatively, the inductances can be computed numerically from the variation of magnetic energy. One way of expressing the total magnetic field energy in the magnetic problems is via the integral:

$$W = \frac{1}{2} \int \mathbf{A} \cdot \mathbf{J} \ dV$$

The FEM calculations have been performed at various combination of the $(i,g)$ values, in the whole range of their change. It is possible to perform magnetostatic simulation of the inductor, or to analyse time dependent variable field problem, by introducing corresponding values of excitation current in the both cases. As the field solution has been obtained, magnetic quantities of interest can be computed.

Series of computational results are well established basis for accomplishing numerical Finite Element Analysis (FEA). The magnetizing inductance of the inductor can be calculated in several ways, by using different numerical approaches based on the FEM results.

A. Once the postprocessor of the software FEMM has been run, the magnetising inductance can be derived from the magnetic energy equation.

It can be noted that another different expression for magnetic field energy in a system with just one circuit, at a specific working point defined with a pair $(i_o,\lambda_o)$, can be derived in a form:

$$W = \frac{1}{2} L_{m} i_o^2$$

Setting these two expressions for energy equal to one another and solving for $L_{m}$ yields:

$$L_{m} = \frac{\int \mathbf{A} \cdot \mathbf{J} \ dV}{i_o^2}$$

For the example problem, $J$ is nonzero only inside the coils, so the $\mathbf{A} \cdot \mathbf{J}$ integral need only be taken over the coils. The result of performing the calculation is:

$$L_{m} = 0.2925 \ \text{[H]}$$

B. An alternative approach is to obtain stored energy via another integral:

$$W = \frac{1}{2} \int B \cdot H \ dV$$

where this integral is taken over the entire problem domain instead of just over the coils and is directly computed from energy integration.

The magnetizing inductance is derived from Eq. (15):

$$L_{m} = \frac{2W}{i^2}$$

For the considered problem, the result of calculation yields to the value:

$$L_{m} = 0.2905 \ \text{[H]}$$

C. Finally, using postprocessing results of the numerical FEM computation, we calculate the flux linkage:

$$\lambda = \frac{\int \mathbf{A} \cdot \mathbf{J} dS}{i}$$

Direct implementation of the Flux Linkage/Current of the winding coil unambiguously is interpreted as magnetizing inductance.

It is not likely that inductance obtained from Eq. (21) in a nonlinear problem will be the result sought; but, as we use nonlinear FEM solver at a specified working point $(i_o,\lambda_o)$ the...
calculated value is quite accurate, and the FEM result is:

\[ I_m = 0.2986 \text{ [H]} \]  

(22)

**Inductance characteristics**

In continuation, on the basis of the computational FEM results, the magnetising inductance characteristics \( I_m = f(g,i) \) will be derived. It is requested to run program FEMM by changing gap length \( g \) and winding current \( i \). The energy concept (A) from the previous heading, as an approach to magnetising inductance calculation, is selected.

On the basis of computational FEM results, the \( I_m = f(i,g) \) characteristics are derived; they are presented in Fig. 5.

![Fig. 5. Magnetising inductance characteristics \( I_m = f(i,g) \)](image)

From an analysis of the characteristics, it is evident that greater air-gap length means linear magnetic field, yielding to almost constant magnetising inductance, independent on the excitation current.

**Comparative analysis**

The exact value of the inductance can be reached only by measurements. After testing of the considered E-I inductor \([1,8]\) it is obtained the relevant result for the main inductance \( L = 0.4023 \text{ [H]} \). Using the calculated value of the leakage inductance \( L_k \), the magnetizing inductance is:

\[ I_{value} = 0.2825 \text{ [H]} \]  

(23)

The computed results with proposed methodologies are compared with the measured results at operating conditions with same input voltage and/or current.

Comparative analysis presented in Table 1, is the best way for evaluation of different approaches for calculation of inductance.

**Table 1 Comparative results of magnetizing inductance \( I_m \) [H]**

<table>
<thead>
<tr>
<th>Analytical results</th>
<th>Numerical FEM results</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear case</td>
<td>MCT</td>
<td>A.</td>
</tr>
<tr>
<td>0.3340</td>
<td></td>
<td>0.2703</td>
</tr>
<tr>
<td>Nonlinear case</td>
<td></td>
<td>0.2837</td>
</tr>
</tbody>
</table>

Obviously the linear calculations of the magnetising inductance are not enough reliable; their only advantage is the simplicity. The result is erroneous for +18.23%.

In Table 1 is presented the nonlinear analytical result obtained by iterative procedure proposed by the authors.

The improvement of the accuracy is evident and the computational error is only +0.42%; but it should be emphasised that the procedure is rather complicated.

Application of the magnetic circuit theory (MCT) gives satisfactory results; the calculated result differs from the measured one for −4.32%.

Calculations based on the finite element results, by using three different methodologies A, B and C are quite correct. The computational error is +3.54%, +2.83% and +4.71%, respectively; as expected the flux linkage/current method is the least accurate. But in general, the numerical FEM approach is the most accurate and reliable.

**Conclusion**

The aim of the present paper is to show how inductance is calculated by using various possibilities of Finite Element Method (FEM) simulations and to compare these results to the approximation obtained through analytical and magnetic circuit approach. All obtained results are compared with the experimentally measured value. The studied structure is an iron core inductor with variable air gap.

For rough estimation of the inductance it is suggested to start with analytical approach. The magnetising inductance can be easily calculated by using simple magnetic circuit theory, although this approach typically is not enough accurate as ignores flux leakage and fringing effects. The best approach is to apply Finite Element Method.

Consequently, the magnetising inductances obtained from the FEM simulations are always higher than those analytically computed; this is essentially due to the flux lines external to the air-gap, as can be seen in Fig. 2. The numerical results are always closer to the measured values, proving the FEM calculations as the most accurate.

**References**


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